## 1. Kinematika pohybu hmotného bodu

1.1 $v = \dfrac{s}{\dfrac{s}{3v_1} + \dfrac{2s}{3v_2}} = \dfrac{3v_1 v_2}{2v_1 + v_2} \Rightarrow v_1 = \dfrac{v v_2}{3v_2 - 2v} = 18\ \mathrm{km \cdot h^{-1}}$

1.2 $v = at = \dfrac{2s}{t^2} \cdot t = \dfrac{2s}{t} = 140\ \mathrm{m \cdot s^{-1}}$

1.3 $\Delta v = g(t_2 - t_1) = 10\ \mathrm{m \cdot s^{-1}},\ s = \dfrac{1}{2} g\left(t_2^2 - t_1^2\right) = 25\ \mathrm{m}$

1.4 $5v_0 = v_0 + at,\ s = v_0 t + \dfrac{1}{2} at^2 \Rightarrow v_0 = \dfrac{s}{3t} = 5\ \mathrm{m \cdot s^{-1}},\ a = \dfrac{4s}{3t^2} = 4\ \mathrm{m \cdot s^{-2}}$

1.5 $v = v_0 - at,\ s = v_0 t - \dfrac{1}{2} at^2 \Rightarrow a = \dfrac{v_0^2 - v^2}{2s} = 1{,}4\ \mathrm{m \cdot s^{-2}},\ t = \dfrac{2s}{v_0 + v} = 7{,}7\ \mathrm{s}$

1.6 $\sin \alpha' = \dfrac{5{,}5}{1{,}5} \Rightarrow \alpha' = 16°,\ \alpha = 106°,\ v = \sqrt{5{,}5^2 - 1{,}5^2}\ \mathrm{m \cdot s^{-1}} = 5{,}3\ \mathrm{m \cdot s^{-1}}$

1.7 $v = \pi d_1 f_1 = \pi d_2 f_2 = 7{,}5\ \mathrm{m \cdot s^{-1}},\ f_2 = f_1 \dfrac{d_1}{d_2} = 6{,}0\ \mathrm{Hz}$

1.8 $T = 86\,164\ \mathrm{s},\ a_\mathrm{d} = R \cos\varphi \cdot \dfrac{4\pi^2}{T^2},\ a_{\mathrm{d}1} = 0{,}024\ \mathrm{m \cdot s^{-2}},\ a_{\mathrm{d}2} = 0{,}017\ \mathrm{m \cdot s^{-2}}$

1.9 $\dfrac{mv^2}{\left(\dfrac{d}{2} - h\right)} = mg \Rightarrow v = \sqrt{\mathrm{g}\left(\dfrac{d}{2} - h\right)} = 5{,}4\ \mathrm{m \cdot s^{-1}}$

## 2. Dynamika pohybu hmotného bodu

2.1 $a = \dfrac{v}{t} = \dfrac{m_0 g}{2m + m_0} \Rightarrow m = \dfrac{m_0 (gt - v)}{2v} = 0{,}78\ \mathrm{kg}$

2.2 $F = \dfrac{mv^2}{2s} = 750\ \mathrm{kN}$

2.3 $a = g(\sin\alpha - f \cos\alpha) = 1{,}9\ \mathrm{m \cdot s^{-2}}$

2.4 $a = \dfrac{\left| m_1 \dfrac{h}{l} - m_2 \right|}{m_1 + m_2} g = 0{,}23\ \mathrm{m \cdot s^{-2}}$

2.5 a) $F_1 = mg = 49\ \mathrm{N}$, b) $F_2 = m(g + a) = 59\ \mathrm{N}$, c) $F = m(g - a) = 42\ \mathrm{N}$

2.6 $k = \dfrac{mg - \dfrac{mv^2}{r}}{mg} = 1 - \dfrac{v^2}{gr} = \dfrac{1}{3}$

2.7 a) $F_\mathrm{s} = \dfrac{mv^2}{r} = 120\ \mathrm{N}$, b) $\mathrm{tg}\,\alpha = \dfrac{F_\mathrm{s}}{F_\mathrm{G}} = \dfrac{v^2}{gr} \Rightarrow \alpha = 11°$, c) $F_\mathrm{t} \geq F_\mathrm{s} \Rightarrow f = \dfrac{v^2}{gr} = 0{,}20$

## 3. Mechanická práce, mechanická energie, výkon

3.1 $\eta = \dfrac{mgv}{P} = 0{,}95$

3.2 $P = \dfrac{\eta m g h}{t} = \dfrac{\eta \rho V g h}{t} = \eta \rho Q_\mathrm{V} g h \Rightarrow h = \dfrac{P}{\eta \rho Q_\mathrm{V} g} = 17\ \mathrm{m}$

3.3 $t = \dfrac{\rho S h g\left(H - \dfrac{h}{2}\right)}{\eta P_0} = 230\,\text{min}$

3.4 $mgh = mg \cdot 2r + \dfrac{1}{2}mv^2, \dfrac{mv^2}{r} = mg \;\Rightarrow\; h = \dfrac{5}{2}r = 0{,}50\ \text{m}$

3.5 $v = \sqrt[3]{\dfrac{P}{k}} = 30\ \text{m}\cdot\text{s}^{-1}$

3.6 $v = \dfrac{m_1}{m_1 + m_2}v_1 = \dfrac{5}{8}v_1 = 0{,}25\ \text{m}\cdot\text{s}^{-1},\ \Delta E = \dfrac{m_1 m_2}{2(m_1 + m_2)}v_1^2 = 1\,050\ \text{J},\ \dfrac{\Delta E}{E_1} = \dfrac{m_2}{m_1 + m_2} = \dfrac{3}{8} = 0{,}375$

3.7 $v_1 = \dfrac{m_1 + m_2}{m_1}\sqrt{2gh} = 230\ \text{m}\cdot\text{s}^{-1}$

## 4. Gravitační pole a pohyby v něm

4.1 $y(2) = h + v_0 t - \dfrac{1}{2}gt^2 = 6{,}4\ \text{m},\ t_\text{d} = \dfrac{v_0 + \sqrt{v_0^2 + 2gh}}{g} = 2{,}4\ \text{s},\ v_\text{d} = \sqrt{v_0^2 + 2gh} = 18\ \text{m}\cdot\text{s}^{-1}$

4.2 $x(2) = v_0 t_1 = 40\ \text{m},\ y(2) = h - \dfrac{1}{2}gt_1^2 = 60\ \text{m},\ t_d = \sqrt{\dfrac{2h}{g}} = 4{,}0\ \text{s},\ v_d = \sqrt{v_0^2 + 2gh} = 44\ \text{m}\cdot\text{s}^{-1}$

4.3 $mr\omega^2 = \dfrac{GmM}{r^2},\ \omega = \dfrac{2\pi}{T} \;\Rightarrow\; M = \dfrac{4\pi^2 r^3}{GT^2} = 2{,}0\cdot 10^{30}\ \text{kg}$

4.4 $\rho = \dfrac{M}{\dfrac{4}{3}\pi R^3},\ a_g = \dfrac{F_g}{m} = \dfrac{GM}{R^2} \;\Rightarrow\; M = \dfrac{a_g R^2}{G} = 6{,}0\cdot 10^{24}\ \text{kg},\ \rho = \dfrac{3a_g}{4\pi GR} = 5\,500\ \text{kg}\cdot\text{m}^{-3}$

4.5 $a_\text{g} = \dfrac{GM}{(3R)^2} = \dfrac{1}{9}\dfrac{GM}{R^2} = \dfrac{1}{9}a_{\text{g}0} = 1{,}09\ \text{m}\cdot\text{s}^{-2},\ v_\text{k} = \sqrt{\dfrac{GM}{3R}} = \dfrac{1}{\sqrt{3}}\sqrt{\dfrac{GM}{R}} = \dfrac{1}{\sqrt{3}}v_{\text{k}0} = 4\,600\ \text{m}\cdot\text{s}^{-1}$

4.6 $\dfrac{T^2}{T_\text{Z}^2} = \dfrac{a^3}{a_\text{Z}^3},\ 2a = r_\text{p} + r_\text{a} \;\Rightarrow\; r_\text{a} = 2a - r_\text{p} = 2a\sqrt[3]{\dfrac{T^2}{T_\text{Z}^2}} - r_\text{p} = 35{,}3\ \text{au},\ r_\text{a}v_\text{a} = r_\text{p}v_\text{p} \;\Rightarrow\; \dfrac{v_\text{p}}{v_\text{a}} = \dfrac{r_\text{a}}{r_\text{p}} = 59:1$

## 5. Mechanika tuhého tělesa

5.1 $M = 2Fr\sin\alpha = 18\ \text{N}\cdot\text{m}$

5.2 $F_\text{ř}\dfrac{d}{2} = F\cdot 2r \;\Rightarrow\; F_\text{ř} = \dfrac{4r}{d}F = 2{,}5\ \text{kN},\ W = 2F\cdot 2\pi r\cdot 20 = 1\,900\ \text{J}$

5.3 $W = mg\left(\dfrac{\sqrt{2}a}{2} - \dfrac{a}{2}\right) = \dfrac{\sqrt{2}-1}{2}\rho a^4 g = 135\ \text{J},\ F_\text{min}\cdot\sqrt{2}a = mg\cdot\dfrac{a}{2} \;\Rightarrow\; F_\text{min} = \dfrac{\rho a^3 g}{2\sqrt{2}} = 580\ \text{N}$

5.4 $F_1 + F_2 = mg,\ F_1 r_1 = F_2 r_2 \;\Rightarrow\; F_1 = \dfrac{r_2}{r_1 + r_2}mg = 500\ \text{N},\ F_2 = \dfrac{r_1}{r_1 + r_2}mg = 400\ \text{N}$

5.5 $m_1 g\left(\dfrac{l}{2} - d\right) = m_2 gd \;\Rightarrow\; m_2 = m_1\left(\dfrac{l}{2d} - 1\right) = 65\,\text{kg}$

5.6 $Fl\cos\alpha = mg\dfrac{l}{2}\sin\alpha \;\Rightarrow\; F = \dfrac{mg}{2}\text{tg}\,\alpha = 17\ \text{N}$

5.7 $m$ v počátku, $2m$ na ose $x$, $4m$ na ose $y$ $\;\Rightarrow\; x_\text{T} = \dfrac{1}{2}d = 0{,}50\ \text{m},\ y_\text{T} = \dfrac{7}{10}d = 0{,}70\ \text{m}$

5.8 $T = \pi\sqrt{\frac{2J}{Fs}} = 0{,}17\ \mathrm{s}$

5.9 $f = \frac{1}{\pi}\sqrt{\frac{Pt}{2J}} = 50\ \mathrm{Hz}$, $r = \sqrt[5]{\frac{2J}{\pi\rho}} = 0{,}48\ \mathrm{m}$

## 6. Mechanika kapalin a plynů

6.1 $\rho_\mathrm{t} abcg = \rho abhg \ \Rightarrow\ h = \frac{\rho_\mathrm{t}}{\rho}c = 0{,}7c = 7\ \mathrm{cm}$

6.2 $W = \rho_\mathrm{t} Shgh - \rho Shg\frac{h}{2} = Sh^2 g\left(\rho_\mathrm{t} - \frac{\rho}{2}\right) = 72\ \mathrm{kJ}$

6.3 $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2, \frac{\pi d_1^2}{4}v_1 = \frac{\pi d_2^2}{4}v_2 \ \Rightarrow\ \Delta p = p_2 - p_1 = \frac{1}{2}\rho v_1^2\left(1 - \frac{d_1^4}{d_2^4}\right) = -4\,800\ \mathrm{Pa}$

6.4 $d = 2\sqrt{\frac{Q_\mathrm{V}}{\pi v}} = 5{,}6\ \mathrm{mm}$

6.5 $v = \sqrt{2gh} = 21\,\mathrm{m\cdot s^{-1}}, P = Fv = pSv = \frac{1}{2}\rho v^2 \cdot \frac{\pi d^2}{4}\cdot v = \frac{1}{8}\pi d^2\rho v^3 = \frac{\sqrt{2}}{4}\pi d^2\rho\sqrt{g^3h^3} = 8{,}5\,\mathrm{MW}$

nebo $P = \frac{E_\mathrm{k}}{t} = \frac{\frac{1}{2}mv^2}{t} = \frac{\frac{1}{2}\rho V v^2}{t} = \frac{1}{2}\rho Q_\mathrm{V} v^2 = \frac{1}{2}\rho\cdot Sv\cdot v^2 = \frac{1}{2}\rho\cdot\frac{\pi d^2}{4}\cdot v^3 = \ldots$

6.6 $P = F_\mathrm{odp}v = \frac{1}{2}CS\rho v^3 = 22\ \mathrm{kW}$

## 7. Základní poznatky molekulárně kinetické teorie látek

7.1 a) o 45 K, 279 K a 234 K

7.2 a) 108 g, b) $1{,}8\cdot10^{-25}\ \mathrm{kg}$, c) $5{,}6\cdot10^{21}$, d) $6{,}022\cdot10^{23}$

7.3 a) $M_\mathrm{r} = 201, M_\mathrm{m} = 201\ \mathrm{g\cdot mol^{-1}}$, b) $m_\mathrm{m} = M_\mathrm{r}m_\mathrm{u} = 3{,}34\cdot10^{-27}\,\mathrm{kg}$

c) $n = \frac{m}{M_\mathrm{m}} = 0{,}184\,\mathrm{mol}, N = nN_\mathrm{A} = 1{,}11\cdot10^{23}$

7.4 a) $m = \rho V = 0{,}12\,\mathrm{kg}$, b) $n = \frac{m}{M_\mathrm{m}} = 60\,\mathrm{mol}$,

c) $N = nN_\mathrm{A} = 3{,}6\cdot10^{25}$, d) $m_\mathrm{m} = 2A_\mathrm{r}m_\mathrm{u} = 3{,}32\cdot10^{-27}\ \mathrm{kg}$

7.5 a) $n = \frac{N}{N_\mathrm{A}} = 83\,\mathrm{mol}$, b) $m = N\cdot 2A_\mathrm{r}m_\mathrm{u} = 2{,}3\,\mathrm{kg}$, c) $V = nV_\mathrm{m} = 1{,}86\ \mathrm{m^3}$

7.6 $W = nN_\mathrm{A}\cdot E_0 = 0{,}50\ \mathrm{MJ}$

7.7 a) $\frac{2}{32} = 0{,}063$, b) $\frac{10}{32} = 0{,}31$, $\frac{20}{32} = 0{,}63$

## 8. Struktura a vlastnosti plynů

8.1 $V_\mathrm{m} = 22{,}4\,\mathrm{l\cdot mol^{-1}}, N = nN_\mathrm{A} = \frac{V}{V_\mathrm{m}}N_\mathrm{A} = 2{,}7\cdot10^{22} \ \Rightarrow\ t = 3\cdot10^{16}\,\mathrm{s} = 9\cdot10^{8}\ \mathrm{r}$

8.2 $Q = 2{,}4\,\mathrm{kJ}, W = -1{,}5\,\mathrm{kJ}, \Delta U = Q + W = 0{,}9\,\mathrm{kJ}$, teplota vzrostla

8.3 $\frac{p_1}{T_1} = \frac{1{,}2p_1}{T_1 + 200\,\mathrm{K}} \ \Rightarrow\ T_1 = 1\,000\,\mathrm{K}, T_2 = 1\,200\,\mathrm{K}$

8.4 $pV = \frac{m}{M_\mathrm{m}} RT \;\Rightarrow\; p = \frac{\rho RT}{M_\mathrm{m}} = 110\,000\ \mathrm{Pa}$

8.5 $\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} \;\Rightarrow\; \frac{1500 \cdot V_1}{288} = \frac{\{p_2\} \cdot 0{,}25 V_1}{318} \;\Rightarrow\; p_2 = 6\,600\ \mathrm{Pa}$

8.6 $pV = \frac{m}{M_\mathrm{m}} RT \;\Rightarrow\; m = \frac{pVM_\mathrm{m}}{RT} = 8{,}4\ \mathrm{g}$

8.7 $p_1 V_1^\kappa = p_2 \left(\frac{V_1}{15}\right)^\kappa \;\Rightarrow\; p_2 = 15^\kappa p_1 = 4{,}4\ \mathrm{MPa}, \frac{p_1 V_1}{T_1} = \frac{15^\kappa p_1 \frac{V_1}{15}}{T_2} \;\Rightarrow\; T_2 = 15^{\kappa-1} T_1 = 890\ \mathrm{K}$

## 9. Termodynamika

9.1 $mgh = mc\Delta t \;\Rightarrow\; \Delta t = \frac{gh}{c} = 0{,}14\ ^\circ\mathrm{C}$

9.2 $\frac{1}{2} m v^2 = mc\Delta t \;\Rightarrow\; \Delta t = \frac{v^2}{2c} = 150\ ^\circ\mathrm{C}$

9.3 $m_1 c(t_1 - t) = m_2 c(t - t_2) \;\Rightarrow\; m_1 = \frac{t - t_2}{t_1 - t} m_2 = 1{,}75\ \mathrm{kg}$

9.4 $m_1 c_1 (t_1 - t) = m_2 c_2 (t - t_2) \;\Rightarrow\; t_2 = t - \frac{m_1 c_1 (t_1 - t)}{m_2 c_2} = 12\ ^\circ\mathrm{C}$

9.5 a) $C = m_1 c_1 = 330\ \mathrm{J \cdot K^{-1}}$, b) $m_1 c_1 (t - t_1) = m_2 c_2 (t_2 - t) \;\Rightarrow\; t = \frac{m_1 c_1 t_1 + m_1 c_1 t_1}{m_1 c_1 + m_1 c_1} = 73\ ^\circ\mathrm{C}$

9.6 a) $\eta P_0 \tau = mc\Delta t \;\Rightarrow\; \eta = \frac{mc\Delta t}{P_0 \tau} = 0{,}80$, b) $E = P_0 \tau = 506\ \mathrm{kJ} = 0{,}14\ \mathrm{kWh} \;\Rightarrow$ cena 1,4 Kč

9.7 $m_1 c_1 (t_1 - t) = m_2 c_2 (t - t_2) \;\Rightarrow\; c_2 = \frac{m_1 (t_1 - t)}{m_2 (t - t_2)} c_1 = 400\ \mathrm{J \cdot kg^{-1} \cdot K^{-1}}$

## 10. Struktura a vlastnosti pevných látek a kapalin

10.1 $l_1 (1 + \alpha_1 \Delta t) = l_2 (1 + \alpha_2 \Delta t) \;\Rightarrow\; \Delta t = \frac{l_1 - l_2}{l_1 \alpha_1 - l_2 \alpha_2} = 530\ ^\circ\mathrm{C}$

10.2 $F = \sigma_\mathrm{p} \cdot 20\pi \frac{d^2}{4} = 38\,000\ \mathrm{N}$

10.3 Voda: $V = V_1 (1 + \beta \Delta t) = 0{,}491 < 0{,}501$, navíc i vnitřní objem hrnku se zvětší

10.4 $m = \rho_1 V_1 = \rho V = \rho_1 \frac{V}{1 + 3\alpha\Delta t} = 8{,}7\ \mathrm{kg}$

10.5 $\Delta p = E \frac{\Delta l}{l_1} = E \frac{l_1 \alpha \Delta t}{l_1} = E \alpha \Delta t = 120\ \mathrm{MPa}$

10.6 $\sigma = \frac{mg}{2l} = 0{,}039\ \mathrm{N \cdot m^{-1}}$

10.7 $\frac{2\sigma_1}{R} = h_1 \rho_1 g, \frac{2\sigma_2}{R} = h_2 \rho_2 g \;\Rightarrow\; \sigma_2 = \frac{h_2}{h_1} \frac{\rho_2}{\rho_1} \sigma_1 = 0{,}49\ \mathrm{N \cdot m^{-1}}$

10.8 $h\rho g = \frac{2\sigma}{R} \;\Rightarrow\; m = \rho \cdot \pi R^2 h = \rho \cdot \pi R^2 \cdot \frac{2\sigma}{R\rho g} = \frac{2\pi R\sigma}{g} = 12\ \mathrm{mg}$

## 11. Skupenské přeměny látek

11.1 $Q = mc\Delta t + ml_{\mathrm{t}} = 267\ \mathrm{kJ}$

11.2 $\tau = \dfrac{m_1 c\Delta t + m_2 l_{\mathrm{v}}}{P} = 500\ \mathrm{s}$

11.3 $\dfrac{mc\Delta t}{\tau_1} = \dfrac{ml_{\mathrm{v}}}{\tau_2} \ \Rightarrow\ \tau_2 = \tau_1 \dfrac{l_{\mathrm{v}}}{c\Delta t} = 51\,\mathrm{min}$

11.4 $(m_1 c + C)(t_1 - t) = m_2 l_{\mathrm{t}} + m_2 ct \ \Rightarrow\ t = \dfrac{(m_1 c + C)t_1 - m_2 l_{\mathrm{t}}}{(m_1 + m_2)c + C} = 45\,^{\circ}\mathrm{C}$

11.5 $Q_1 = m_1 c_1 t_1 = 420\ \mathrm{kJ}, Q_2 = m_2 c_2 t_2 = 37{,}8\ \mathrm{kJ}, L_{\mathrm{t}} = m_2 l_{\mathrm{t}} = 400{,}8\ \mathrm{kJ} \ \Rightarrow\ Q_1 < Q_2 + L_{\mathrm{t}}$,

neroztátý led $m_1' = \dfrac{Q_2 + L_{\mathrm{t}} - Q_1}{l_{\mathrm{t}}} = 0{,}056\,\mathrm{kg}$

11.6 $m_1 c(t - t_1) = \eta m_2 l_{\mathrm{v}} + m_2 c(t_2 - t) \ \Rightarrow\ t = \dfrac{m_2(\eta l_{\mathrm{v}} + ct_2) + m_1 c_1 t_1}{(m_1 + m_2)c} = 72\,^{\circ}\mathrm{C}$

## 12. Kmitavý pohyb

12.1 $v_{\mathrm{m}} = y_{\mathrm{m}} \cdot 2\pi f = 0{,}55\ \mathrm{m \cdot s^{-1}}$, $a_{\mathrm{m}} = y_{\mathrm{m}} \cdot (2\pi f)^2 = 1\,500\ \mathrm{m \cdot s^{-2}}$

12.2 $y = 0{,}002 \sin\left(50\pi\{t\} + \dfrac{\pi}{2}\right)\mathrm{m}$, $t = \dfrac{3}{4}T = 0{,}030\ \mathrm{s}$, $v_{\mathrm{m}} = y_{\mathrm{m}}\omega = 0{,}31\ \mathrm{m \cdot s^{-1}}$

12.3 a) $y_{\mathrm{m}} = 5\,\mathrm{cm}$, $T = 4\,\mathrm{s}$, $\varphi_0 = \dfrac{\pi}{4}\,\mathrm{rad}$, b) $\Delta t = \dfrac{T}{8} = 0{,}5\ \mathrm{s}$, c) $y(t_1) = 3{,}5\,\mathrm{cm}$, $y(t_2) = 0\,\mathrm{cm}$

12.4 $f = 1{,}25\ \mathrm{Hz} \ \Rightarrow\ m = \dfrac{k}{4\pi^2 f^2} = 4{,}1\,\mathrm{kg}$, $\Delta l = \dfrac{mg}{k} = 0{,}16\ \mathrm{m}$

12.5 $k = \dfrac{F}{\Delta l} = 260\ \mathrm{N \cdot m^{-1}}$, $T = 2\pi\sqrt{\dfrac{m}{k}} = 0{,}55\ \mathrm{s}$

12.6 $W = \dfrac{1}{2}ky^2, F = ky \ \Rightarrow\ k = \dfrac{F^2}{2W} = 32{,}5\ \mathrm{N \cdot m^{-1}}$,

$y = 0{,}04 \sin\left(4\pi\{t\} - \dfrac{\pi}{2}\right)\mathrm{m} = 0{,}04 \sin\left(4\pi\{t\} + \dfrac{3\pi}{2}\right)\mathrm{m}$

12.7 $v = y_{\mathrm{m}}\omega\cos\dfrac{\pi}{3} = y_{\mathrm{m}}\sqrt{\dfrac{k}{m}}\cos\dfrac{\pi}{3} = 0{,}25\ \mathrm{m \cdot s^{-1}}$, $F_{\mathrm{m}} = ky_{\mathrm{m}} = 1{,}3\,\mathrm{N}$

## 13. Mechanické vlnění, akustika

13.1 $y = 0{,}04\,\mathrm{m} \cdot \sin 2\pi\left(\dfrac{t}{2} - \dfrac{x}{1{,}5}\right)$, $\varphi = 10\pi$ (proběhlo 5 kmitů)

13.2 $\dfrac{\Delta\varphi}{2\pi} = \dfrac{\Delta x}{\lambda}, \lambda = \dfrac{v}{f} \ \Rightarrow\ \Delta\varphi = \dfrac{\Delta x}{\lambda} \cdot 2\pi = \dfrac{\Delta x f}{v} \cdot 2\pi = \dfrac{3\pi}{2}\,\mathrm{rad}$

13.3 a) $\dfrac{\Delta\varphi}{2\pi} = \dfrac{\Delta x}{\lambda} \ \Rightarrow\ \lambda = \dfrac{2\pi}{\Delta\varphi} \cdot \Delta x = 1{,}6\ \mathrm{m}$, b) $f = \dfrac{v}{\lambda} = 210\ \mathrm{Hz}$

13.4 $\lambda = cT = 5\,\mathrm{m} \ \Rightarrow$

$y = y_{\mathrm{m}} \sin 2\pi\left(\dfrac{t}{T} - \dfrac{x}{\lambda}\right) = 2\sin 2\pi\left(\dfrac{10}{0{,}125} - \dfrac{28{,}75}{5}\right)\mathrm{cm} = 2\sin 148{,}5\pi\ \mathrm{cm} = 2{,}0\ \mathrm{cm}$

13.5 $\Delta x = \dfrac{\lambda}{2} = \dfrac{c}{2f} = 0{,}39\ \mathrm{m}$, $d = \dfrac{\Delta x}{2} = 0{,}19\ \mathrm{m}$

13.6 $l = 3 \cdot \frac{\lambda}{2} \Rightarrow f = \frac{c}{\lambda} = \frac{3c}{2l} = 750\ \text{Hz}$

## 14. Elektrické pole, kondenzátory

14.1 $E = \frac{F}{|q|} = 300\ \text{V} \cdot \text{m}^{-1}, E_0 = E \cdot \left(\frac{r}{R}\right)^2 = 7\,500\ \text{V} \cdot \text{m}^{-1},\ E = k\frac{Q}{r^2} \Rightarrow Q = \frac{Er^2}{k} = 8{,}3\ \text{nC}$

14.2 $\vec{E} = \vec{E}_1 + \vec{E}_2,\ E = E_1 = E_2 = k\frac{|Q_1|}{a^2} = 1800\ \text{V} \cdot \text{m}^{-1}$, směr shodný se směrem od $Q_2$ do $Q_1$

14.3 $F = e\frac{|\varphi_2 - \varphi_1|}{d} = 6{,}4 \cdot 10^{-16}\ \text{N}$, směr $\overrightarrow{\text{BA}}$, $W = e|\varphi_2 - \varphi_1| = 480\,\text{eV} = 7{,}7 \cdot 10^{-17}\ \text{J}$

14.4 $C = \frac{3}{2}C_1 = 0{,}30\ \mu\text{F},\ Q_1 = Q_2 = C_1\frac{U}{2} = 2{,}4\ \mu\text{C},\ Q_3 = C_1 U = 4{,}8\ \mu\text{C}$

14.5 $C = \varepsilon_0 \varepsilon_\text{r} \frac{S}{d} = 2{,}1\ \text{nF}$

14.6 $Q = C_1 U_1 = C_2 U_2 \Rightarrow U_2 = \frac{C_1}{C_2}U_1 = \frac{\varepsilon_0 \frac{S}{d_1}}{\varepsilon_0 \frac{S}{d_2}}U_1 = \frac{d_2}{d_1}U_1 = 92\ \text{V}$,

$$\Delta E = \frac{1}{2}Q(U_1 - U_2) = \frac{1}{2}\varepsilon_0 \frac{S}{d_1}U_1\left(U_1 - \frac{d_2}{d_1}U_1\right) = \frac{\varepsilon_0 S U_1^2 (d_1 - d_2)}{2d_1^2} = 5{,}6 \cdot 10^{-8}\ \text{J}$$

## 15. Obvod stejnosměrného proudu

15.1 $I_1 = 1{,}5\ \text{A}, I_2 = 0{,}9\ \text{A}, I_3 = 0{,}6\ \text{A}, U_1 = 45\ \text{V}, U_2 = U_3 = 36\ \text{V}$

15.2 $P = RI^2, U_\text{e} = (R + R_\text{i})I \Rightarrow R_\text{i} = U_\text{e}\sqrt{\frac{R}{P}} - R = 1{,}4\ \Omega$

15.3 $\rho_1 = \frac{\rho}{1 + \alpha\Delta t} = \frac{RS}{l(1 + \alpha\Delta t)} = 0{,}017\ \mu\Omega\text{m}$

15.4 $R = \frac{U}{I} = \frac{1\,000}{7}\ \Omega,\ R_{12} = R - R_3 = \frac{300}{7}\ \Omega,\ R_2 = \left(\frac{1}{R_{12}} - \frac{1}{R_1}\right)^{-1} = 60\ \Omega$

15.5 $U = U_e - R_i I = 4{,}2\ \text{V},\ R = \frac{U_e}{I} - R_i = 8{,}4\ \Omega,\ I_\text{max} = \frac{U_e}{R_i} = 7{,}5\ \text{A}$

15.6 $P = \frac{mc\Delta t}{\tau} = 1\,400\ \text{W},\ R = \frac{U^2}{P} = 38\ \Omega$

15.7 $I_1 = \frac{P_1}{P_1 + P_2}I = 3{,}2\ \text{A},\ I_2 = \frac{P_2}{P_1 + P_2}I = 0{,}8\ \text{A}$

## 16. Elektrický proud v látkách a ve vakuu

16.1 $R = \frac{U}{I} = \frac{UM_\text{m}t}{mF\nu} = 3{,}0\ \Omega$

16.2 $m = \frac{1}{F}\frac{M_\text{m}}{\nu}It = 0{,}34\ g$

16.3 a) $A = \frac{M_\text{m}}{F\nu} = 3{,}29 \cdot 10^{-7}\ \text{kg} \cdot \text{C}^{-1}$, b) $h = \frac{m}{\rho S} = \frac{AIt}{\rho S} = 0{,}48\ \text{mm}$

16.4 $m = \rho Sh = \dfrac{M_\mathrm{m} Q}{F\nu}, E = QU \;\Rightarrow\; S = \dfrac{M_\mathrm{m} E}{F\nu U\rho h} = 0{,}57\ \mathrm{m}^2$

16.5 $d = \dfrac{1}{2}at^2 = \dfrac{v^2}{2a}, a = \dfrac{qE}{m} \;\Rightarrow\; d = \dfrac{mv^2}{2qE} = 1{,}2\cdot 10^{-12}\ \mathrm{m}$

16.6 $\dfrac{1}{2}mv^2 = eU \;\Rightarrow\; v = \sqrt{\dfrac{2eU}{m}} = 1{,}9\cdot 10^7\ \mathrm{m\cdot s^{-1}}$

16.7 $W = eU = eEd = 3{,}2\cdot 10^{-16}\ \mathrm{J} = 2\,000\ \mathrm{eV}$

## 17. Magnetické pole

17.1 $\dfrac{\mu_0}{2\pi}\dfrac{I_1}{x} = \dfrac{\mu_0}{2\pi}\dfrac{I_2}{d-x} \;\Rightarrow\; x = \dfrac{I_1}{I_1+I_2}d = \dfrac{3}{5}d = 3{,}0\ \mathrm{cm}$

17.2 $B = \dfrac{F}{Il} = 0{,}080\ \mathrm{T}$

17.3 $F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{d}l = 4{,}0\cdot 10^{-4}\ \mathrm{N}$

17.4 $60° \to 90°, B = \dfrac{F_2 - F_1}{Il(\sin 90° - \sin 60°)} = 0{,}90\ \mathrm{T}$

17.5 $B = \dfrac{2ky}{Il} = 1{,}2\ \mathrm{T}$

17.6 $B = \mu\dfrac{N}{l}I \;\Rightarrow\; \dfrac{N}{l} = \dfrac{B}{\mu_\mathrm{r}\mu_0 I} = 1800\ \mathrm{m^{-1}}, l = 17\ \mathrm{cm}$

17.7 $F_\mathrm{m} = Bev = 1{,}9\cdot 10^{-15}\ \mathrm{N}, \; r = \dfrac{mv}{Be} = 0{,}19\ \mathrm{m}$

17.8 $E = Bv = 2\,000\ \mathrm{V\cdot m^{-1}}$

17.9 $m = \dfrac{Be}{2\pi f} = 1{,}7\cdot 10^{-27}\ \mathrm{kg}$, proton

## 18. Elektromagnetická indukce

18.1 $v = \dfrac{|U_\mathrm{i}|}{Bl} = 0{,}75\ \mathrm{m\cdot s^{-1}}$

18.2 $U_{\mathrm{i}1} = -L\dfrac{\Delta I}{\Delta t_1} = -25\ \mathrm{V}, \; U_{\mathrm{i}2} = -L\dfrac{\Delta I}{\Delta t_2} = 150\ \mathrm{V}$

18.3 $I = \dfrac{Blv}{R} = 4{,}0\cdot 10^{-7}\ \mathrm{A}$ (po dobu $t = \dfrac{l}{v} = 0{,}25\ \mathrm{s}$)

18.4 Normála smyčky změnila úhel ze 60° na 30°: $U_\mathrm{i} = -\dfrac{B\pi r^2(\cos 30° - \cos 60°)}{\Delta t} = -0{,}072\ \mathrm{V}$

18.5 $N = -\dfrac{U_\mathrm{i}\Delta t}{\Delta\Phi} = 100$

18.6 a) $\Phi = NBS = N\cdot\mu_0\dfrac{N}{l}I\cdot S = \dfrac{\mu_0 N^2 S}{l}\cdot I = LI \;\Rightarrow\; L = \dfrac{\mu_0 N^2 S}{l} = 6{,}8\cdot 10^{-4}\ \mathrm{H}$

b) $E_\mathrm{m} = \dfrac{1}{2}LI^2 = 1{,}4\ \mathrm{mJ}$

## 19. Obvod střídavého proudu

19.1 $X_{\mathrm{C}} = \dfrac{1}{2\pi f C} = \dfrac{U}{I} \;\Rightarrow\; C = \dfrac{I}{2\pi f U} = 2{,}1\ \mu\mathrm{F}$

19.2 $R = X_{\mathrm{C}} = \dfrac{1}{2\pi f C} \;\Rightarrow\; f = \dfrac{1}{2\pi R C} = 66\ \mathrm{kHz}$

19.3 $\dfrac{X_{\mathrm{L2}}}{X_{\mathrm{L1}}} = \dfrac{L\cdot 2\pi f_2}{L\cdot 2\pi f_1} = \dfrac{f_2}{f_1} \;\Rightarrow\; f_2 = \dfrac{X_{\mathrm{L2}}}{X_{\mathrm{L1}}} f_1 = 200\ \mathrm{Hz}$

19.4 a) $X_{\mathrm{C}} = \dfrac{1}{2\pi f C} = \dfrac{U}{I} \;\Rightarrow\; f = \dfrac{I}{2\pi C U} = 1{,}1\ \mathrm{kHz}$

b) $\dfrac{I_1}{I} = \dfrac{2\pi f_1 C U}{2\pi f C U} = \dfrac{f_1}{f} \;\Rightarrow\; I_1 = \dfrac{f_1}{f} I = 14\ \mathrm{mA}$, c) $I_{\mathrm{ss}} = 0$

19.5 $I_{\mathrm{ss}} = \dfrac{U}{R} = 600\ \mathrm{mA}$, $I = \dfrac{U}{\sqrt{R^2 + X_{\mathrm{L}}^2}} = \dfrac{U}{\sqrt{R^2 + (2\pi f L)^2}} = 19\ \mathrm{mA}$

19.6 $Z = \sqrt{R^2 + \left(2\pi f L - \dfrac{1}{2\pi f C}\right)^2} = 950\ \Omega$

## 20. Střídavý proud v energetice

20.1 $P = UI\cos\varphi \;\Rightarrow\; \cos\varphi = \dfrac{P}{UI} \;\Rightarrow\; \varphi = 61°$

20.2 $P = U_1 I_1 = k U_1 I_2 \;\Rightarrow\; U_1 = \dfrac{P}{k I_2} = 180\ \mathrm{V}$

20.3 $\dfrac{N_2}{N_1} = \dfrac{U_2}{U_1} = \dfrac{U_2}{U_1}\cdot\dfrac{I_1}{I_1} = \dfrac{U_2 I_1}{P} \;\Rightarrow\; N_2 = \dfrac{U_2 I_1}{P} N_1 = 16$

20.4 a) $\dfrac{N_2}{N_1} = \dfrac{U_2}{U_1} \;\Rightarrow\; N_2 = \dfrac{U_2}{U_1} N_1 = 21$

b) $P = U_1 I_1 = U_2 I_2 \;\Rightarrow\; I_1 = \dfrac{P}{U_1} = 0{,}17\ \mathrm{A}$, $I_2 = \dfrac{P}{U_2} = 1{,}7\ \mathrm{A}$

20.5 $P = 3\cdot\dfrac{\left(\sqrt{3}U\right)^2}{R} \;\Rightarrow\; R = \dfrac{9U^2}{R} = 56\ \Omega$

20.6 $P_{\mathrm{z}} = R_{\mathrm{v}} I^2 = R_{\mathrm{v}} \dfrac{P^2}{U^2} = 37\ \mathrm{W}$

## 21. Optické soustavy a optické zobrazení

21.1 $a' = 16\,\mathrm{cm},\ y' = 51\,\mathrm{cm}$ (přesně $a' = \dfrac{300}{19}\,\mathrm{cm},\ y' = 50\,\mathrm{cm}$)

21.2 $Z = 0{,}18,\ a' = -8{,}2\,\mathrm{cm}$

21.3 $Z = -\dfrac{1}{2} \Rightarrow a = 3f = 36\,\mathrm{cm}, \left(Z = \dfrac{1}{2} \Rightarrow a = -f = -12\,\mathrm{cm}\ \text{neexistuje}\right)$

21.4 $f = -10\,\mathrm{cm},\ a' = -6\,\mathrm{cm}$

21.5 $a_1 = 60\,\mathrm{cm},\ a_1' = 30\,\mathrm{cm};\ a_2 = 30\,\mathrm{cm},\ a_2' = 60\,\mathrm{cm}$

21.6 $a' = -18\,\mathrm{cm},\ f = -45\,\mathrm{cm}$, zdánlivý

## 22. Vlnová optika, elektromagnetické spektrum

22.1 $\sin\alpha_m = \dfrac{1}{n} \;\Rightarrow\; \alpha_m = 36° < 45°$, nastane

22.2 $n_2 = \dfrac{c}{v_2} = 1{,}37,\ \sin\alpha_1 = \dfrac{n_2}{n_1}\sin\alpha_2 \Rightarrow \alpha_1 = 51°,\ \lambda_2 = \lambda_1\dfrac{n_1}{n_2} = 400\,\text{nm}$

22.3 $\sin\alpha_2 = \dfrac{1}{n}\sin\alpha_1 \Rightarrow \alpha_2 = 32{,}95\ °,\ \beta_1 = \varphi - \alpha_2 = 27{,}05\ °,$

$\sin\beta_2 = n\sin\beta_1 \Rightarrow \beta_2 = 36{,}24°, \delta = \alpha_1 + \beta_2 - \varphi = 21°$

22.4 $2nd + \dfrac{\lambda}{2} = k\lambda \Rightarrow \lambda_{\max} = \dfrac{4nd}{2k\text{-}1} = 750\ \text{nm}\ (k=1),$

$2nd + \dfrac{\lambda}{2} = (2k+1)\dfrac{\lambda}{2} \Rightarrow \lambda_{\min} = \dfrac{2nd}{k} = 375\ \text{nm}\ (k=1)$, mimo světelný obor

22.5 $b\sin\alpha = \lambda \;\Rightarrow\; b = 2\,645\ \text{nm} \;\Rightarrow\; \dfrac{1}{b} = 380\ \text{mm}^{-1}$

22.6 $\sin\alpha = \dfrac{\lambda}{b} \doteq \dfrac{y}{d} \;\Rightarrow\; y_1 = 2{,}00\ \text{cm},\ y_2 = 3{,}75\ \text{cm},\ \Delta y = 1{,}75\ \text{cm}$

## 23. Speciální teorie relativity

23.1 $l = l_0\sqrt{1-\dfrac{v^2}{c^2}} \;\Rightarrow\; v = c\sqrt{1-\left(\dfrac{l}{l_0}\right)^2} = 0{,}14c$

23.2 $\Delta m = \dfrac{mc_1\Delta t}{c^2} = 4{,}7\cdot10^{-12}\ \text{kg}$

23.3 $l_0 = 0{,}5\ \text{m}$ ... klidová délka v soust. laboranta, $t = 1{,}7\cdot10^{-9}\ \text{s}$ ... relativistický čas v soust. laboranta

$l$ ... relativistická délka v soust. protonu, $t_0 = ?$ ... vlastní čas v soust. protonu

$v = \dfrac{l_0}{t} = \dfrac{0{,}5\ \text{m}}{1{,}7\cdot10^{-9}\ \text{s}} = 2{,}94\cdot10^8\ \text{m}\cdot\text{s}^{-1}, t_0 = t\sqrt{1-\dfrac{v^2}{c^2}} = 3{,}3\cdot10^{-10}\ \text{s}$

obecně $v = \dfrac{l_0}{t} = \dfrac{l}{t_0} \;\Rightarrow\; t_0 = \dfrac{l}{l_0}t = \dfrac{l_0\sqrt{1-\dfrac{v^2}{c^2}}}{l_0}t = t\sqrt{1-\dfrac{l_0^2}{t^2c^2}} = \sqrt{t^2-\dfrac{l_0^2}{c^2}} = 3{,}3\cdot10^{-10}\ \text{s}$

nebo $v = \dfrac{l_0}{t},\ t_0 = t\sqrt{1-\dfrac{v^2}{c^2}} = t\sqrt{1-\dfrac{l_0^2}{t^2c^2}} = \sqrt{t^2-\dfrac{l_0^2}{c^2}} = 3{,}3\cdot10^{-10}\ \text{s}$

23.4 Soustava průvodčího: $l = 0{,}75c\cdot\Delta t_0 = 1{,}125\cdot10^8\ \text{m},\ l_0 = \dfrac{l}{\sqrt{1-\dfrac{v^2}{c^2}}} = 1{,}7\cdot10^8\ \text{m}$

Soustava výpravčího: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-\dfrac{v^2}{c^2}}} = 0{,}756\,\text{s},\ l_0 = 0{,}75c\cdot\Delta t = 1{,}7\cdot10^8\ \text{m}$

23.5 $v_1 = c\sqrt{1-\left(\dfrac{l_1}{l_0}\right)^2} = 0{,}624c,\ v_2 = c\sqrt{1-\left(\dfrac{l_2}{l_0}\right)^2} = 0{,}511c \;\Rightarrow\; u = \dfrac{v_1+v_2}{1+\dfrac{v_1v_2}{c^2}} = 0{,}86c$

23.6 $\Delta E = (m_\text{p} + m_\text{n} - m_\text{D})c^2 = 3{,}8\cdot10^{-13}\ \text{J} = 2{,}4\ \text{MeV}$

## 24. Kvantová fyzika, elektronový obal atomu

24.1 $\lambda = \dfrac{h}{mv} = \dfrac{h\sqrt{1-\dfrac{v^2}{c^2}}}{m_0 v} = \dfrac{h\sqrt{0{,}19}}{m_0 \cdot 0{,}9c} = 1{,}2 \cdot 10^{-12}\ \mathrm{m}$

24.2 $E = \dfrac{hc}{\lambda} = 12{,}4\ \mathrm{eV},\ W = \dfrac{hc}{\lambda_m} = 5{,}0\ \mathrm{eV},\ E_e = E - W = 7{,}4\ \mathrm{eV}$

24.3 $\dfrac{hc}{\lambda} = W_v + \dfrac{1}{2}mv^2 \ \Rightarrow\ v = \sqrt{\dfrac{2}{m}\left(\dfrac{hc}{\lambda} - W_v\right)} = 6{,}4 \cdot 10^5\ \mathrm{m \cdot s^{-1}}$

24.4 $mr\omega^2 = k\dfrac{e^2}{r^2},\ \omega = 2\pi f \ \Rightarrow\ f = \dfrac{e}{2\pi}\sqrt{\dfrac{k}{mr^3}} = 6{,}6 \cdot 10^{15}\ \mathrm{Hz},\ I = fe = 1{,}1\ \mathrm{mA}$

24.5 $m_0 c^2 = \dfrac{hc}{\lambda} \ \Rightarrow\ \lambda = \dfrac{h}{m_0 c} = 2{,}4 \cdot 10^{-12}\ \mathrm{m}\ E_{\mathrm{m,n}} = E_{\mathrm{m}} - E_{\mathrm{n}} = E_1\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right) = 13{,}21\,\mathrm{eV},$

$E_{\mathrm{m,n}} = hf_{\mathrm{m,n}} = \dfrac{hc}{\lambda_{\mathrm{m,n}}} \ \Rightarrow\ \lambda_{\mathrm{m,n}} = \dfrac{hc}{E_{\mathrm{m,n}}} = 94\ \mathrm{nm}$

## 25. Fyzika atomového jádra

25.1 $\varepsilon = \dfrac{6m_{\mathrm{p}} + 8m_{\mathrm{n}} - m_{\mathrm{j}}}{14}c^2 = \dfrac{\left(6 \cdot 1{,}6726 + 8 \cdot 1{,}6750 - 13{,}999\ 95 \cdot 1{,}6605\right) \cdot 10^{-27}}{14} \cdot \left(2{,}998 \cdot 10^8\right)^2\ \mathrm{J} =$

$= 7{,}56\ \mathrm{MeV}$

25.2 $\varepsilon = \dfrac{\dfrac{E}{c^2}}{10m_{\mathrm{p}} + 10m_{\mathrm{n}}} = 0{,}043\ \%$

25.3 ${}^{206}_{82}\mathrm{Pb}$

25.4 $\dfrac{N}{N_0} = e^{-\frac{\frac{1}{3}T}{T}\ln 2} = e^{-\frac{\ln 2}{3}} = \left(\dfrac{1}{2}\right)^{\frac{\frac{1}{3}T}{T}} = \left(\dfrac{1}{2}\right)^{\frac{1}{3}} \Rightarrow\ \dfrac{N_0 - N}{N_0} = 1 - \sqrt[3]{0{,}5} = 0{,}794$

25.5 $N = N_0 e^{-\frac{t}{T}\ln 2} \ \Rightarrow\ t = -\dfrac{\ln\dfrac{N}{N_0}}{\ln 2}T = 2\,380\,\mathrm{r}$

25.6 $N = N_0 e^{-\frac{t}{T}\ln 2} \ \Rightarrow\ T = -\dfrac{\ln 2}{\ln 0{,}8}t = 3\,\mathrm{h}\,6\,\mathrm{min}$

25.7 $\dfrac{\Delta m}{m_0} = \dfrac{\Delta E}{235m_{\mathrm{u}}c^2} = 9{,}1 \cdot 10^{-4}$

25.8 $Pt = \eta E,\ \dfrac{m}{235m_{\mathrm{u}}} = \dfrac{E}{E_1} \ \Rightarrow\ m = 235m_{\mathrm{u}}\dfrac{Pt}{\eta E_1} = 960\,\mathrm{kg}$